

## AMENDMENT

### In The Claims

Please amend the claims as follows:

1. (Currently amended) An apparatus for computing multiple integral of a multidimensional integrand function A to be integrated with using a vector map mapping function f with unbounded support which converts an m ( $m \geq 1$ )-dimensional vector having real number components into another m-dimensional vector having real number components, by which a multidimensional density function  $\rho$  for a limiting distribution resulting from repeatedly applying the map vector mapping function f to an m-dimensional vector u is analytically solvable, said apparatus comprising:

a first storage unit which stores an m-dimensional vector  $x = (x_1, x_2, \dots, x_m)$ ;

a second storage unit which stores a scalar value w;

a first computing unit which computes a vector  $x' = f(x) = (x'_1, x'_2, \dots, x'_m)$  resulting from applying said vector map f to said vector x being stored in said first storage unit;

a second computing unit which computes a scalar value  $w' = A(x)/\rho(x)$  based on said vector x being stored in said first storage unit ~~and said scalar value w being stored in said second storage unit~~;

an update unit which updates the value stored in said first storage unit by storing said vector  $x'$  computed by said first computing unit on said first storage unit, and updates the value in said second storage unit by adding said scalar value  $w'$  computed by said second computing unit to a value to be stored in said second storage unit; and

an output unit which computes, when the number of update times by said update unit becomes  $c$  ( $c \geq 1$ ), a scalar value  $s = w/(c+1)$  based on said scalar value  $w$  being stored in said second storage unit when the number of update times by said update unit becomes  $c$  ( $c \geq 1$ ), and outputs said scalar value  $s$  as a result of the multiple integral.

2. (Original) The apparatus according to claim 1, wherein said scalar value stored in said second storage unit first is a result from dividing a value resulting from applying said function  $A$  to said  $m$ -dimensional vector stored in said first storage unit first by a value resulting from applying said density function  $\rho$  to said  $m$ -dimensional vector stored in said first storage unit first.

3. (Original) The apparatus according to claim 1, wherein said scalar value stored in said second storage unit first is 0; and

    said output unit computes a scalar value  $s' = w/c$ , and outputs said scalar value  $s'$  as the result of the multiple integral instead of said scalar value  $s$ .

4. (Original) The apparatus according to claim 1 further comprising:  
    a convergence rate obtainer which obtains convergence rate of scalar values sequentially output by said output unit while varying said number of update times  $c$  for each of plural vector maps  $g_1, g_2, \dots, g_k$  ( $k \geq 2$ ) which are prepared as said vector map  $f$ ;

a vector map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a vector map  $g_h$  ( $1 \leq h \leq k$ ) which shows fastest convergence rate; and an output controller which controls said output unit to output said scalar value with using said vector map  $g_h$  as said vector map  $f$  and the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

5. (Previously amended) The apparatus according to claim 1, wherein a multidimensional density function  $\rho$  representing the limiting distribution of a vector sequence  $u, f(u), f(f(u)), f(f(f(u))), \dots$  resulting from applying said vector map  $f$  to a predetermined  $m$ -dimensional vector  $u = (u_1, u_2, \dots, u_m)$  for equal to or greater than 0 times, satisfies the following property of:

$$\begin{aligned}\rho(v) &= \prod_{i=0}^m \rho_i(u_i); \\ \rho_i(u_i) &\sim c_{-i} |u_i|^{-(1+a)} \text{ for } u_i \rightarrow -\infty; \\ \rho_i(u_i) &\sim c_{+i} |u_i|^{-(1+a)} \text{ for } u_i \rightarrow +\infty; \\ (a > 0, 1 \leq i \leq m, c_{-i} > 0, c_{+i} > 0).\end{aligned}$$

6. (Previously amended) The apparatus according to claim 5, wherein said vector map  $f$  is defined as

$$f(u) = (f_1(u_1), f_2(u_2), \dots, f_m(u_m))$$

by a function  $f_i(t) = g_i(d_i t)/d$  ( $d_i > 0$ ) which is defined in  $1 \leq i \leq m$ , and said map  $g_i$  is defined by any one of the following maps  $\varphi_j$  ( $1 \leq j \leq 8$ ) and a natural number  $n_i$  ( $n_i \geq 2$ ), as follows:

$$g_i(\varphi_j(\theta)) = \varphi_j(n_i(\theta));$$

$$\varphi_1(\theta) = -\operatorname{sgn}(\tan\theta) / |\tan\theta|^{1/a};$$

$$\varphi_2(\theta) = -\operatorname{sgn}(\tan\theta) \times |\tan\theta|^{1/a};$$

$$\varphi_3(\theta) = -\operatorname{sgn}(\cos\theta) / |\tan\theta|^{1/a};$$

$$\varphi_4(\theta) = -\operatorname{sgn}(\cos\theta) \times |\tan\theta|^{1/a};$$

$$\varphi_5(\theta) = \operatorname{sgn}(\cos\theta) / |\tan\theta|^{1/a};$$

$$\varphi_6(\theta) = \operatorname{sgn}(\cos\theta) \times |\tan\theta|^{1/a};$$

$$\varphi_7(\theta) = \operatorname{sgn}(\sin\theta) / |\tan\theta|^{1/a};$$

$$\varphi_8(\theta) = \operatorname{sgn}(\sin\theta) \times |\tan\theta|^{1/a};$$

$$\operatorname{sgn}(t) = 1 \quad \text{for } t > 0;$$

$$\operatorname{sgn}(t) = 0 \quad \text{for } t = 0;$$

$$\operatorname{sgn}(t) = -1 \quad \text{for } t < 0.$$

7. (Currently Amended) The apparatus according to claim 5 further comprising:

a convergence rate obtainer which defines said map  $f$  for each of plural positive numbers

$q_1, q_2, \dots, q_3 q_k$  ( $k \geq 2$ ) prepared as an invariable  $a$ , and obtains convergence rates of the scalar

values sequentially output by said output unit while varying said number of update times  $c$ ;

a positive number selector which refers the convergence rates obtained by said convergence rate obtainer, and selects an integer  $q_h$  ( $1 \leq h \leq k$ ) which shows the fastest convergence rate; and

an output controller which defines said map  $f$  with using said positive number  $q_h$  said invariable  $a$ , and controls said output unit to output said scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

8. (Original) The apparatus according to claim 6 further comprising:
  - a convergence rate obtainer which defines said map  $g_i$  with using plural ones of said maps  $\varphi_j$ , and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times  $c$ ;
  - a map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects one of said maps  $\varphi_j$  which shows the fastest convergence rate; and
  - an output controller which defines said map  $g_i$  with using said map  $\varphi_j$  selected by said map selector, and controls said output unit to output the scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

9. (Original) The apparatus according to claim 6 further comprising:
  - a convergence rate obtainer which defines said map  $g_i$  relating to each of plural natural numbers  $P_1, P_2, \dots, P_k$  ( $k \geq 2$ ) as said natural numbers  $n_i$ , and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times  $c$ ;

a natural number selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a natural number  $P_h$  ( $1 \leq h \leq k$ ) which shows the fastest convergence rate; and

an output controller which defines said natural number map  $g_i$  with using said natural number  $P_h$  as said natural number  $n_i$ , and controls said output unit to output the scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

10. (Original) The apparatus according to claim 1, wherein said output unit computes said scalar value  $s$  each time said update unit carries out update, compares said latest scalar value  $s$  with the former scalar value which is computed at former update, and outputs said latest scalar value  $s$  if a result of the comparison satisfies a predetermined condition for terminating the computation.

11. (Currently amended) A method for computing multiple integral of a multidimensional integrand function  $A$  to be integrated with using a vector map mapping function  $f$  with unbounded support which converts an  $m$  ( $m \geq 1$ ) -dimensional vector having real number components into another  $m$ -dimensional vector having real number components, by which a multidimensional density function  $\rho$  for a limiting distribution resulting from repeatedly applying the map vector mapping function  $f$  to an  $m$ -dimensional vector  $a$  is analytically solvable, said method using

a first storage unit which stores an  $m$ -dimensional vector  $x = (x_1, x_2, \dots, x_m)$ , and

a second storage unit which stores a scalar value  $w$ ,  
said method comprising the steps of:  
computing a vector  $x' = f(x) = (x'_1, x'_2, \dots, x'_m)$  resulting from applying said vector map  
 $f$  to said vector  $x$  being stored in said first storage unit;  
computing a scalar value  $w' = A(x)/\rho(x)$  based on said vector  $x$  being stored in said first  
storage unit ~~and said scalar value  $w$  being stored in said second storage unit~~;  
updating the value stored in said first storage unit by storing said vector  $x$  computed ~~by~~  
~~said first computing unit~~ on said first storage unit, and updating the value in said second storage  
unit by adding said scalar value  $w'$  computed ~~by said second computing unit~~ to a value to be  
stored on said second storage unit; and  
computing, when the number of update times by said update unit becomes  $c$  ( $c \geq 1$ ), a  
scalar value  $s = w/(c+1)$  based on said scalar value  $w$  being stored in said second storage unit  
when the number of update times by said update unit becomes  $c$  ( $c \geq 1$ ), and outputting said scalar  
value  $s$  as a result of the multiple integral.

12. (Original) The method according to claim 11, wherein said scalar value stored in  
said second storage unit first is a result from dividing a value resulting from applying said  
function  $A$  to said  $m$ -dimensional vector stored in said first storage unit first by a value resulting  
from applying said density function  $\rho$  to said  $m$ -dimensional vector stored in said first storage  
unit first.

13. (Original) The method according to claim 11, wherein said scalar value stored in said second storage unit first is 0, and

    said outputting step computes a scalar value  $s' = w/c$ , and outputs said scalar value  $s'$  as the result of the multiple integral instead of said scalar value  $s$ .

14. (Currently Amended) The method according to claim 11 further comprising the steps of:

    obtaining convergence rate of scalar values sequentially output by said outputting step while varying said number of update times  $c$  for each of plural vector maps  $g_1, g_2, \dots, g_k$  ( $k \geq 2$ ) which are prepared as said vector map  $f$ ; and

    referring to the convergence rates obtained by said convergence rate obtainner, and selecting a vector map  $g_h$  ( $1 \leq h \leq k$ ) which shows fastest convergence rate, and  
    said outputting step outputs said scalar value with using said vector map  $g_h$  as said vector map  $f$  and the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

15. (Previously amended) The method according to claim 11, wherein a multidimensional density function  $\rho$  representing the limiting distribution of a vector sequence  $u, f(u), f(f(u)), f(f(f(u))), \dots$

    resulting from applying said vector map  $f$  to a predetermined  $m$ -dimensional vector  $u = (u_1, u_2, \dots, u_m)$  for equal to or greater than 0 times, satisfies the following property of:

$$\rho(u) = \prod_{i=0}^m \rho_i(u_i);$$

$$\rho_i(u_i) \sim c_{-i} |u_i|^{-(1+a)} \text{ for } u_i \rightarrow -\infty;$$

$$\rho_i(u_i) \sim c_{-i} |u_i|^{-(1+a)} \text{ for } u_i \rightarrow +\infty;$$

$$(a>0, 1 \leq i \leq m, c_{-i}>0, c_{+i}>0);$$

16. (Previously amended) The method according to claim 15, wherein said vector map  $f$  is defined as

$$f(u) = (f_1(u_1), f_2(u_2), \dots, f_m(u_m))$$

by a function  $f_i(t) = g_i(d_i t) / d_i$  ( $d_i > 0$ ) which is defined in  $1 \leq i \leq m$ , and said map  $g_i$  is defined by any one of the following maps  $\varphi_j$  ( $1 \leq j \leq 8$ ) and a natural number  $n_i$ , ( $n_i \geq 2$ ) as follows:

$$g_i \varphi_j(\theta) = \varphi_j(n_i \theta);$$

$$\varphi_1(\theta) = -\text{sgn}(\tan \theta) / |\tan \theta|^{1/a};$$

$$\varphi_2(\theta) = -\text{sgn}(\tan \theta) \times |\tan \theta|^{1/a};$$

$$\varphi_3(\theta) = -\text{sgn}(\cos \theta) / |\tan \theta|^{1/a};$$

$$\varphi_4(\theta) = -\text{sgn}(\cos \theta) \times |\tan \theta|^{1/a};$$

$$\varphi_5(\theta) = \text{sgn}(\cos \theta) / |\tan \theta|^{1/a};$$

$$\varphi_6(\theta) = \text{sgn}(\cos \theta) \times |\tan \theta|^{1/a};$$

$$\varphi_7(\theta) = \text{sgn}(\sin \theta) / |\tan \theta|^{1/a};$$

$$\varphi_8(\theta) = \text{sgn}(\sin \theta) \times |\tan \theta|^{1/a};$$

$$\text{sgn}(t) = 1 \text{ for } t > 0;$$

$$\text{sgn}(t) = 0 \text{ for } t = 0;$$

$$\text{sgn}(t) = -1 \text{ for } t < 0.$$

17. (Original) The method according to claim 15 further comprising the step of:  
defining said map  $f$  for each of plural positive numbers  $q_1, q_2, \dots, q_k$  ( $k \geq 2$ ) prepared as  
an invariable  $a$ , and obtaining convergence rates of the scalar values sequentially output by said  
output unit while varying said number of update times  $c$ ;  
referring to the obtained convergence rates, and selecting a positive number  $q_h$  ( $1 \leq h \leq k$ )  
which shows the fastest convergence rate; and  
defining said map  $f$  with using said positive number  $q_h$  as said invariable  $a$ , and  
controlling output of said scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of  
said number of update times  $c$ .

18. (Original) The method according to claim 16 further comprising the steps of:  
defining said map  $g_i$  with using plural ones of said maps  $\varphi_j$ , and obtaining convergence  
rates of the scalar values sequentially output by said output step while varying said number of  
update times  $c$ ;  
referring to the obtained convergence rates, and selecting one of said maps  $\varphi_j$  which  
shows the fastest convergence rate; and  
defining said map  $g_i$  with using said selected map  $\varphi_j$  selected, and controlling output of  
the scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update  
times  $c$ .

19. (Original) The method according to claim 16 further comprising the steps of:  
defining said map  $g_i$  relating to each of plural natural numbers  $P_1, P_2, \dots, P_k$  ( $k \geq 2$ ) as  
said natural numbers  $n_i$ , and obtaining convergence rates of the scalar values sequentially output  
by said outputting step while varying said number of update times  $c$ ;  
referring to the obtained convergence rates, and selecting a natural number  $P_h$  ( $1 \leq h \leq k$ )  
which shows the fastest convergence rate; and  
defining said natural number map  $g_i$  with using said natural number  $P_h$ , as said natural  
number  $n_i$ , and controlling output of the scalar values with using the number of update times  
 $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

20. (Original) The method according to claim 11, wherein said outputting step  
computes said scalar value  $s$  each time said updating step carries out update, compares said latest  
scalar value  $s$  with the former scalar value which is computed at former update, and outputs said  
latest scalar value  $s$  if a result of the comparison satisfies a predetermined condition for  
terminating the computation.

21. (Previously amended) A computer readable recording medium storing a program  
for computing multiple integral of a multidimensional integrand function  $A$  to be integrated with  
using a vector map mapping function  $f$  with unbounded support which converts an  $m$  ( $m \geq 1$ ) -  
dimensional vector having real number components into another  $m$ -dimensional vector having  
real number components, by which a multidimensional density function  $\rho$  for a limiting

distribution resulting from repeatedly applying the map vector mapping function  $f$  to an  $m$ -dimensional vector  $u$  is analytically solvable, said program comprising instructions for:

storing an  $m$ -dimensional vector  $x = (x_1, x_2, \dots, x_m)$  in a first storage unit;

storing a scalar value  $w$  in a second storage unit;

computing a vector  $x' = f(x) = (x'_1, x'_2, \dots, x'_m)$ , using a first computing unit, the vector  $x'$  resulting from applying said vector map  $f$  to said vector  $x$  being stored in said first storage unit;

computing a scalar value  $w' = A(x)/\rho(x)$ , using a first computing unit, the scalar value  $w'$  based on said vector  $x$  being stored in said first storage unit ~~and said scalar value w being stored in said second storage unit~~;

updating the value stored in said first storage unit, using an update unit, by storing said vector  $x'$  computed by said first computing unit on said first storage unit, and updates the value in said second storage unit by adding said scalar value  $w'$  computed by said second computing unit to a value to be stored on said second storage unit; and

computing a scalar value  $s = w/(c+1)$ , when the number of update times by said update unit becomes  $c$  ( $c \geq 1$ ), the scalar value  $s$  based on said scalar value  $w$  being stored in said second storage unit when the number of update times by said update unit becomes  $c$  ( $c \geq 1$ ), and outputs said scalar values as a result of the multiple integral.

22. (Original) The recording medium according to claim 21, wherein said scalar value stored in said second storage unit first is a result from dividing a value resulting from applying said function  $A$  to said  $m$ -dimensional vector stored in said first storage unit first by a value

resulting from applying said density function  $\rho$  to said m-dimensional vector stored in said first storage unit first.

23. (Original) The recording medium according to claim 21, wherein said scalar value stored in said second storage unit first is 0; and

    said output unit computes a scalar value  $s' = w/c$ , and outputs said scalar value  $s'$  as the result of the multiple integral instead of said scalar value  $s$ .

24. (Original) The recording medium according to claim 21, wherein said program further causes said computer to function as:

    a convergence rate obtainer which obtains convergence rate of scalar values sequentially output by said output unit while varying said number of update times  $e$  for each of plural vector maps  $g_1, g_2, \dots, g_k$  ( $k \geq 2$ ) which are prepared as said vector map  $t$

    a vector map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a vector map  $g_h$  ( $1 \leq h \leq k$ ) which shows fastest convergence rate; and

    an output controller which controls said output unit to output said scalar value with using said vector map  $g_h$ , as said vector map  $f$  and the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

25. (Previously amended) The recording medium according to claim 21, wherein a multidimensional density function  $\rho$  representing the limiting distribution of a vector sequence

u,  $f(u)$ ,  $f(f(u))$ ,  $f(f(f(u)))$ ,

resulting from applying said vector map  $f$  to a predetermined m-dimensional vector

$u = (u_1, u_2, \dots, u_m)$  for equal to or greater than 0 times, satisfies the following property of:

$$\rho(u) = \prod_{i=0}^m \rho_i(u_i);$$

$$\rho_i(u_i) \sim c_{-i} |u_i|^{-(1+a)} \text{ for } u_i \rightarrow -\infty;$$

$$\rho_i(u_i) \sim c_{-i} |u_i|^{-(1+a)} \text{ for } u_i \rightarrow +\infty;$$

$$(a > 0, 1 \leq i \leq m, c_{-i} > 0, c_{+i} > 0);$$

26. (Previously amended) The recording medium according to claim 25, wherein said

vector map  $f$  is defined as:

$$f(u) = (f_1(u_1), f_2(u_2), \dots, f_m(u_m))$$

by a function  $f_i(t) = g_i(d_i t)/d_i$  ( $d_i > 0$ ) which is defined in  $1 \leq i \leq m$ , and said map  $g_i$  is

defined by any one of the following maps  $\varphi_j$  ( $1 \leq j \leq 8$ ) and a natural number  $n_i$  ( $n_i \geq 2$ ) as follows:

$$g_i(\varphi_j(\theta)) = \varphi_j(n_i \theta);$$

$$\varphi_1(\theta) = -\text{sgn}(\tan \theta) / |\tan \theta|^{1/a};$$

$$\varphi_2(\theta) = -\text{sgn}(\tan \theta) \times |\tan \theta|^{1/a};$$

$$\varphi_3(\theta) = -\text{sgn}(\cos \theta) / |\tan \theta|^{1/a};$$

$$\varphi_4(\theta) = -\text{sgn}(\cos \theta) \times |\tan \theta|^{1/a};$$

$$\varphi_5(\theta) = \text{sgn}(\cos \theta) / |\tan \theta|^{1/a};$$

$$\varphi_6(\theta) = \text{sgn}(\cos \theta) \times |\tan \theta|^{1/a};$$

$$\varphi_7(\theta) = \text{sgn}(\sin \theta) / |\tan \theta|^{1/a};$$

$$\varphi_8(\theta) = \operatorname{sgn}(\sin\theta) \times |\tan\theta|^{1/a};$$

$$\operatorname{sgn}(t) = 1 \quad \text{for } t > 0;$$

$$\operatorname{sgn}(t) = 0 \quad \text{for } t = 0;$$

$$\operatorname{sgn}(t) = -1 \quad \text{for } t < 0.$$

27. (Original) The recording medium according to claim 25, wherein said program further causes said computer to function as:

a convergence rate obtainer which defines said map  $f$  for each of plural positive numbers  $q_1, q_2, \dots, q_k$  ( $k \geq 2$ ) prepared as an invariable  $a$ , and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times  $c$ ;

an integer selector which refers the convergence rates obtained by said convergence rate obtainer, and selects a positive number  $q_h$  ( $1 \leq h \leq k$ ) which shows the fastest convergence rate; and

an output controller which defines said map  $f$  with using said positive number  $q_h$ , as said invariable  $a$ , and controls said output unit to output said scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

28. (Original) The recording medium according to claim 26, wherein said program further causes said computer to function as:

a convergence rate obtainer which defines said map  $g_i$  with using plural ones of said maps  $\varphi_j$  and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times  $c$ ;

a map selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects one of said maps  $\varphi_j$  which shows the fastest convergence rate; and an output controller which defines said map  $g_i$ , with using said map  $\varphi_j$  selected by said map selector, and controls said output unit to output the scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

29. (Original) The recording medium according to claim 26, wherein said program further causes said computer to function as:

a convergence rate obtainer which defines said map  $g_1$  relating to each of plural natural numbers  $P_1, P_2, \dots, P_k$  ( $k \geq 2$ ) as said natural numbers  $n_i$ , and obtains convergence rates of the scalar values sequentially output by said output unit while varying said number of update times  $c$ ;

a natural number selector which refers to the convergence rates obtained by said convergence rate obtainer, and selects a natural number  $P_h$  ( $1 \leq h \leq k$ ) which shows the fastest convergence rate; and

an output controller which defines said natural number map  $g_i$  with using said natural number  $P_h$  as said natural number  $n_i$ , and controls said output unit to output the scalar values with using the number of update times  $c'$  ( $c' > c$ ) instead of said number of update times  $c$ .

30. (Original) The recording medium according to claim 21, wherein said output unit computes said scalar value  $s$  each time said update unit carries out update, compares said latest

scalar value  $s$  with the former scalar value which is computed at former update, and outputs said latest scalar value  $s$  if a result of the comparison satisfies a predetermined condition for terminating the computation.

31. (Original) The recording medium according to claim 21, wherein said recording medium is a compact disc, a floppy disk, a hard disk, a magneto-optical disk, a digital versatile disc, a magnetic tape, or a semiconductor memory.